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## Bayesian Symbolic Regression and Other Similar Methods as a Tool for Forecasting Commodities Prices

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### Abstract

*Bayesian Symbolic Regression (BSR) is used to predict spot prices of 56 commodities. BSR is a certain improvement over the symbolic regression technique based on genetic programming. Besides, there has been limited applications of the symbolic regression to forecasting prices in economics and finance. Contrary to prior simulations of BSR with synthetic data, this study discusses an application to the real-world data derived from commodities markets. In particular, forecasting one month ahead spot prices of 56 commodities. Indeed, BSR presents valuable capabilities for addressing the complexities associated with variable selection in econometric modelling. It is expected to also handle also some other challenges smoothly. Therefore, this study is carefully tailored to deal with commodity markets time-series data. Moreover, several alternative techniques are also tested, i.e., the symbolic regression with genetic programming, Dynamic Model Averaging, LASSO and RIDGE regressions, time-varying parameters regression, ARIMA, and no-change method, etc. In particular, the main aim is to focus on forecast accuracy. The obtained outcomes can give valuable insights for both researchers and practitioners interested in implementing BSR in econometric and financial projects in the future.*

**Keywords:** Bayesian econometrics, commodity prices, model uncertainty, time-series forecasting, variable selection.

**JEL Classification:** C32, G17, Q02.

### 1. Introduction

The objective of this work was to forecast one month ahead spot prices of 56 commodities with Bayesian Symbolic Regression (BSR) and some other methods dealing with variable uncertainty. In particular, 19 potentially important explanatory variables were considered. The accuracy of the forecasts obtained was analysed.

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## **2. Problem Statement & Research Questions / Aims of the Research**

Two main obstacles emerge when forecasting commodity spot prices. First, the multitude of factors influencing these prices (such as supply and demand dynamics, exchange rates, financial market interactions, speculative pressures, various indices of uncertainty, etc.) challenges the selection of important variables for constructing econometric models like multilinear regression (Gargano and Timmermann, 2014). The second challenge stems from the temporal variability in the impact of specific drivers on commodity prices, suggesting the use of time-varying model parameters and structures (Huang et al., 2021). Additionally, the model's functional form should account for non-linear effects and the intricate nature of the market (Caginalp & Desantis, 2011).

Symbolic regression captures these conundrums. It starts with a set of operators (functions) and employs evolutionary processes such as crossover, mutation, and selection to evolve into a suitable functional form for the model (Koza, 1988). It is a kind of regression analysis, but unlike traditional regression approaches reliant on predefined functional forms (e.g., linear, quadratic), it offers greater flexibility, allowing the method itself to identify the most appropriate mathematical structure and coefficients directly from the data. The algorithm traverses a space of mathematical expressions starting from a population of randomly generated mathematical expressions represented as trees with mathematical operators and variables. These expressions undergo evaluation based on their fitness to the training data, evaluated by a specified fitness function. The algorithm iteratively selects the best evaluated expressions from the current population, and then generates the next population, until a predefined stopping criterion is met (Koza, 1988). Recently, BSR was introduced by Jin et al. (2019). It replaces genetic algorithms with Bayesian symbolic trees, substituting evolutionary processes with Bayesian prior-posterior inference.

The objectives of this study were to compare the performance of forecast accuracy obtained from BRS and competing benchmark methods. Second, we analyse the most desirable specification of the initial parameters for BSR. The main research question was whether BSR generates significantly more accurate forecasts than the benchmark models.

## **3. Data and Research Methods**

Spot prices of 56 commodities, i.e., Brent, Dubai and WTI crude oil, Australian and South African coal, US and European natural gas and Japan liquefied natural gas, cocoa, Arabica and Robusta coffee, Colombo, Kolkata and Mombasa tea, coconut oil, groundnuts, fish meal, palm oil, soybeans, soybean oil, soybean meal, maize, Thai 5% and 10% broken rice, US soft red winter and hard red winter wheat, US bananas, orange, beef, chicken meat, Mexican shrimps, European, US and world sugar, US import tobacco, Cameroon and Malaysian logs, Malaysian sawnwood, plywood, cotton (A index), Singapore traded rubber, phosphate rock, diammonium phosphate, triple superphosphate, urea, potassium chloride, aluminium, iron ore,

copper, lead, tin, nickel, zinc, gold, platinum and silver spot prices were analysed (The World Bank, 2022). Logarithmic differences were inserted into the models.

Similarly as, for example, Gargano and Timmermann (2014) and Drachal (2018), the following explanatory variables were considered: dividend to price ratio, price earnings ratio (Schiller, 2022), US 3-month treasury bills secondary market rate representing short-term rate and US long-term government 10-year bond yields representing long-term rate, term spread, i.e., the difference between the long-term rate of US bonds and US treasury bill rate, default return spread, i.e., the difference between US long-term corporate bonds yield and US treasury bill rate, where long-term corporate bond yield was taken as the index based on bonds with maturities 20-years and above, US Consumer Price Index transformed into logarithmic differences, US industrial production transformed into logarithmic differences, US M1 money stock transformed into logarithmic differences, Kilian global economic activity index (Kilian, 2009), US unemployment rate, real effective exchange rates based on manufacturing Consumer Price Index for US transformed into logarithmic differences, S&P GSCI Commodity Total Return Index transformed into logarithmic differences, US dollar open interest transformed into logarithmic differences, Working's dollar T-index (FRED, 2022; Bloomberg, 2022; Working, 1960; CFTC, 2022), VIX index, i.e., implied volatility based on 30-day S&P 100 index at-the-money options, Global Geopolitical Risk Index, i.e., The Benchmark GPR Index (Caldara & Iacoviello, 2022a; 2022b), MSCI WORLD for developed markets index and MSCI EM for emerging markets index (MSCI, 2022).

The initial data set consisted of 404 monthly observations, beginning on January 1988 and ending on August 2021. The first 100 observations were taken as the in-sample period. Time-series were standardised (i.e., mean was subtracted and outcome divided by standard deviation; both computed over the in-sample period). Explanatory variables were lagged 1 period back.

Computations were done in Python and R (Van Rossum & Drake, 1995; Jin, 2021; R Core Team, 2018) with the help of some additional packages and libraries, for example, implementing benchmark models (Stephens, 2021; Raftery et al., 2010; Onorante et al., 2016; Friedman et al., 2010; Gramacy, 2019; Hastie & Efron, 2013; Hyndman & Khandakar, 2008). All models are listed in Table 1. "Fixed" models are the ones for which parameters were estimated over the in-sample period, and then applied to forecasting over the out-of-sample period. For "recursive" models, rolling estimations with expanding window were done starting from the end of the in-sample period. Benchmark models were taken as by Drachal (2023a), where they are described in detail.

The BSR is fully described by Jin et al. (2019). In short, let  $x_{1,t}, \dots, x_{n,t}$  be the explanatory variables, and let  $y_t$  be the forecasted commodity price (after the mentioned possible transformations). Then,  $y_t = \beta_0 + \beta_1 * f_1(x_{1,1,t-1}, \dots, x_{1,i,t-1}) + \dots + \beta_K * f_k(x_{K,1,t-1}, \dots, x_{K,i,t-1})$ , with  $x_{i,j,t}$  being some of explanatory variables out of  $n = 19$  possible ones which are present in the  $i$ -th component expression, i.e.,  $f_i$ , with  $j = \{1, \dots, n\}$  and  $i = \{1, \dots, K\}$ , is estimated with the Ordinary Least Squares method. The number of components,  $K$ , is specified at the initial stage.  $K = \{1, 2, \dots, 10\}$

were considered. Components  $f_i$  are represented by symbolic trees constructed from a set of operators, such as  $+$ ,  $*$ ,  $1/x$ , etc. In this study, 6 sets were considered. They are presented in Table 2. These sets vary from a very simple one, to the ones capturing non-linear effects, and containing some specific features of economic and financial time-series (Nicolau & Agapitos, 2021; Keijzer, 2004). Prior-posterior inferences were done with parameters' values as suggested by Jin et al. (2019). All possible pairwise values of  $K$  and  $F$  were applied for each commodity during the in-sample period. Then, the pair minimising Root Mean Square Error (RMSE) was chosen for the out-of-sample predictions. GP models were estimated with the same set of operators as indicated by the BSR model for a given commodity.

**Table 1. Estimated models**

| <b>Abbreviation</b> | <b>Description</b>                                                                                                    |
|---------------------|-----------------------------------------------------------------------------------------------------------------------|
| BSR rec             | Bayesian Symbolic Regression (recursive)                                                                              |
| BSR av MSE rec      | Bayesian Symbolic Regression (recursive) with averaging and weights inversely proportional to Mean Square Error (MSE) |
| BSR av EW rec       | Bayesian Symbolic Regression (recursive) with equal weights                                                           |
| GP rec              | Symbolic Regression with Genetic Programming (recursive)                                                              |
| BSR fix             | Bayesian Symbolic Regression (fixed parameters)                                                                       |
| BSR av MSE fix      | Bayesian Symbolic Regression (fixed parameters) with averaging and weights inversely proportional to MSE              |
| BSR av EW fix       | Bayesian Symbolic Regression (fixed parameters) with equal weights                                                    |
| GP fix              | Symbolic Regression with Genetic Programming (fixed parameters)                                                       |
| DMA                 | Dynamic Model Averaging with Occam window                                                                             |
| BMA                 | Bayesian Model Averaging with Occam window                                                                            |
| DMA 1V              | Dynamic Model Averaging over one-variable component models                                                            |
| DMS 1V              | Dynamic Model Selection over one-variable component models                                                            |
| BMA 1V              | Bayesian Model Averaging over one-variable component models                                                           |
| BMS 1V              | Bayesian Model Selection over one-variable component models                                                           |
| LASSO               | LASSO regression (recursive)                                                                                          |
| RIDGE               | RIDGE regression (recursive)                                                                                          |
| EN                  | Elastic net regression (recursive)                                                                                    |
| B-LASSO             | Bayesian LASSO regression (recursive)                                                                                 |
| B-RIDGE             | Bayesian RIDGE regression (recursive)                                                                                 |
| LARS                | Least-angle regression                                                                                                |
| TVP                 | Time-Varying Parameters regression with the forgetting factor equal to 1                                              |
| TVP f               | Time-Varying Parameters regression with the forgetting factor equal to 0.99                                           |
| ARIMA               | Automatic ARIMA (recursive)                                                                                           |
| HA                  | Historical average                                                                                                    |
| NAIVE               | No-change method                                                                                                      |

*Source:* own elaboration.

**Table 2. Operators**

| Abbreviation | Description                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|--------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| F = 1        | $\text{neg}(x_{i,t}) = -x_{i,t}$ ; $\text{add}(x_{i,t},x_{j,t}) = x_{i,t} + x_{j,t}$                                                                                                                                                                                                                                                                                                                                                                  |
| F = 2        | $\text{neg}(x_{i,t}) = -x_{i,t}$ ; $\text{add}(x_{i,t},x_{j,t}) = x_{i,t} + x_{j,t}$ ; $\text{square}(x_{i,t}) = (x_{i,t})^2$                                                                                                                                                                                                                                                                                                                         |
| F = 3        | $\text{neg}(x_{i,t}) = -x_{i,t}$ ; $\text{add}(x_{i,t},x_{j,t}) = x_{i,t} + x_{j,t}$ ; $\text{ma12}(x_{i,t}) = (x_{i,t} + \dots + x_{i,t-11}) / 12$ ; $\text{lag}(x_{i,t}) = x_{i,t-1}$                                                                                                                                                                                                                                                               |
| F = 4        | $\text{neg}(x_{i,t}) = -x_{i,t}$ ; $\text{add}(x_{i,t},x_{j,t}) = x_{i,t} + x_{j,t}$ ; $\text{square}(x_{i,t}) = (x_{i,t})^2$ ; $\text{mul}(x_{i,t},x_{j,t}) = x_{i,t} * x_{j,t}$                                                                                                                                                                                                                                                                     |
| F = 5        | $\text{neg}(x_{i,t}) = -x_{i,t}$ ; $\text{add}(x_{i,t},x_{j,t}) = x_{i,t} + x_{j,t}$ ; $\text{square}(x_{i,t}) = (x_{i,t})^2$ ; $\text{mul}(x_{i,t},x_{j,t}) = x_{i,t} * x_{j,t}$ ; $\text{inv}(x_{i,t}) = 1 / x_{i,t}$ ; $\text{cubic}(x_{i,t}) = (x_{i,t})^3$ ; $\text{sqrt}(x_{i,t}) = \sqrt{x_{i,t}}$ ; $\text{log}(x_{i,t}) = \ln( x_{i,t} )$ ; $\text{ma12}(x_{i,t}) = (x_{i,t} + \dots + x_{i,t-11}) / 12$ ; $\text{lag}(x_{i,t}) = x_{i,t-1}$ |
| F = 6        | $\text{neg}(x_{i,t}) = -x_{i,t}$ ; $\text{add}(x_{i,t},x_{j,t}) = x_{i,t} + x_{j,t}$ ; $\text{lt}(x_{i,t}) = a * x_{i,t} + b$ , with a and b being some real numbers                                                                                                                                                                                                                                                                                  |

Source: own elaboration.

The derived price forecasts were transformed back (from differences to levels) prior to evaluation. RMSE was taken as the primary metric, but outcomes were similar, if Mean Absolute Error or Mean Absolute Scaled Error (Hyndman & Koehler, 2006) were applied.

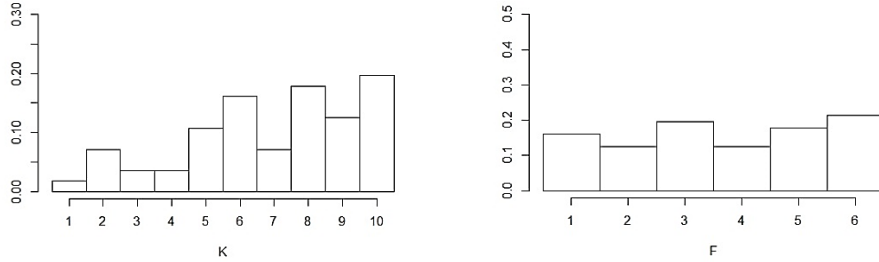
Forecasts from two different models were tested against each other with the Diebold-Mariano test (Diebold & Mariano, 1995; Harvey et al., 1997) and the Giacomini and Rossi fluctuation test (Giacomini & Rossi, 2010). Forecasts from multiple models were tested with the Model Confidence Set (MCS) (Hansen et al., 2011; Bernardi & Catania, 2018). The squared errors loss function was applied to stay in line with the RMSE metric.

In BSR the outcome expression is obtained in the last iteration. Let  $y_1, \dots, y_M$  be the forecasts obtained from M iterations. Let  $w_1, \dots, w_M$  be weights (such that they sum up to 1) ascribed to each of these forecasts. The weighted average forecast is defined as  $w_1 * y_1 + \dots + w_M * y_M$ . In this study, two averaging schemes were applied. The first with weights inversely proportional to MSE-s of the component models. The second, with equal weights for component models. MSE-based weights were divided by the sum of all individual weights in order to sum up to 1 (Stock & Watson, 2004).

#### 4. Findings

Figure 1 reports frequencies of K-s and F-s which minimised RMSE in the in-sample period. It can be seen that, in general, there is a tendency to select higher values of K. Small values are rarely chosen, but the behaviour for higher K-s is quite irregular. Anyways, the most often chosen was K = 10, i.e., the highest considered value. In case of F-s, the most often chosen was F = 6 representing the simplest set of operators expanded with the operator lt. The second most frequently chosen was F = 3 representing the simple set of operators expanded by 12-months moving average and 1<sup>st</sup> lag operators, which are commonly used in economic and financial time-series analysis. This means that the standard functional forms are preferred over complicated formulas.

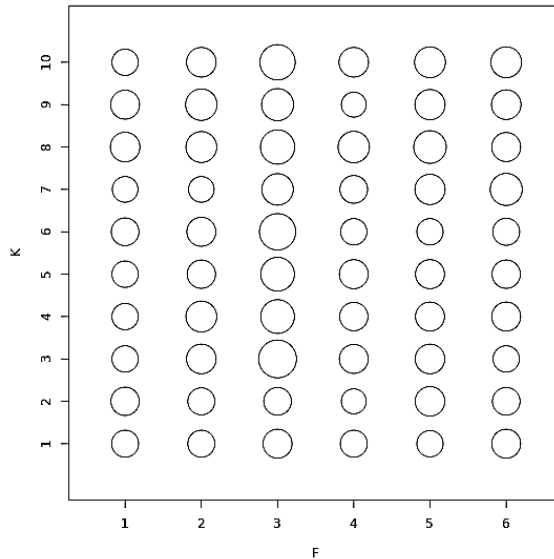
**Figure 1. Histograms of K-s and F-s which minimised RMSE in the in-sample period**



Source: own elaboration.

Figure 2 reports outcomes from the MCS test for various K-s and F-s over the in-sample period. In particular, the radius of a circle is proportional to the frequency (over all commodities) that the model with a given combination of K and F was kept by the MCS test. It can be seen that the most preferred was the combination of K = 3 and F = 3. This means that, despite results reported in Figure 1, smaller values of K can be used without statistically significant difference in forecast accuracy. This can be beneficial for computational costs of the models.

**Figure 2. Outcomes from the MCS test over the in-sample period**



Source: own elaboration.

Table 3 reports frequencies (over all commodities) when the p-values of the Diebold-Mariano tests were less than 5%. The null hypothesis is that the forecasts from both models have the same accuracy. The alternative hypotheses are stated

row-wise in Table 3, due to various tests performed. By the “best” model is understood the one which minimised RMSE for a given commodity out of all models considered (i.e., listed in Table 1). “Model X > Model Y” denotes “The forecast from the Model X has greater accuracy than the forecast from the Model Y”. For the first three rows in Table 3 cases when the “best” model was the same as the competing one were excluded from counting the frequency. Indeed, for 19 commodities, ARIMA minimised RMSE. For 2 commodities it was NAÏVE, and for no commodity it was BSR rec. It can be seen that it is hard to “beat” ARIMA, but quite easier to “beat” NAÏVE. BSR rec does not outperform “best” models often. However, it is also not so often much worse than ARIMA or NAÏVE. Surprisingly, very rarely recursive computations improved forecast accuracy in the case of BSR models. Generally, there is no strong evidence that, in general, recursive computations improved the forecast accuracy over fixed computations. The strongest evidence in favour of recursive computations was found for BSR av EW models. In a reasonably high number of cases BSR rec was found to be more accurate than GP rec.

**Table 3. The Diebold-Mariano test outcomes**

| Alternative hypothesis          | Frequency |
|---------------------------------|-----------|
| “best” > ARIMA                  | 16%       |
| “best” > NAIVE                  | 41%       |
| BSR rec < “best”                | 66%       |
| BSR rec < ARIMA                 | 36%       |
| BSR rec < NAÏVE                 | 34%       |
| BSR rec > BSR fix               | 5%        |
| BSR av MSE rec > BSR av MSE fix | 39%       |
| BSR av EW rec > BSR av EW fix   | 75%       |
| GP rec > GP fix                 | 30%       |
| BSR rec > GP rec                | 45%       |
| BSR fix > GP fix                | 25%       |

Source: own elaboration.

Table 4 reports frequencies of all considered models selected by the MCS test (with 90% confidence level, “TR” statistic and 1000 bootstrap simulations). It can be seen that the most often surviving model was DMA. The second was ARIMA. Other models were kept in less than 50% of the cases. The most often surviving BSR-type model were BSR rec and BSR av MSE rec in 7% of cases each. However, also in 7% of cases, the GP rec was surviving.

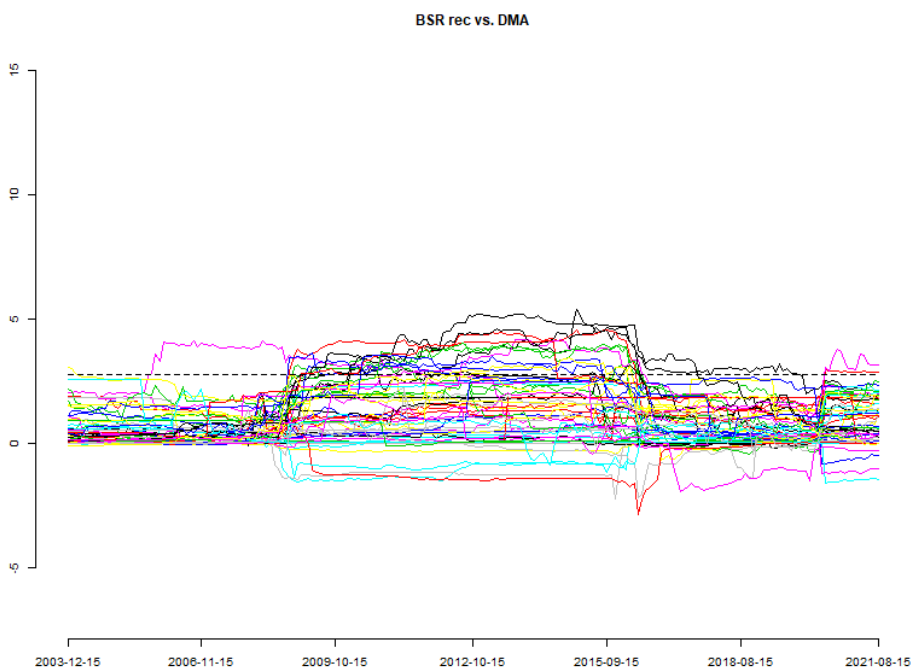
**Table 4. The MCS test outcomes**

| Model   | Frequency | Model          | Frequency |
|---------|-----------|----------------|-----------|
| DMA     | 77%       | BMA 1V         | 5%        |
| ARIMA   | 59%       | RIDGE          | 5%        |
| BMA     | 34%       | BSR fix        | 4%        |
| NAIVE   | 20%       | GP fix         | 4%        |
| B-LASSO | 13%       | EN             | 4%        |
| BMS 1V  | 11%       | BSR av MSE fix | 2%        |
| LASSO   | 11%       | BSR av EW fix  | 2%        |

| Model          | Frequency | Model  | Frequency |
|----------------|-----------|--------|-----------|
| B-RIDGE        | 11%       | DMA 1V | 2%        |
| BSR rec        | 7%        | DMS 1V | 2%        |
| BSR av MSE rec | 7%        | LARS   | 2%        |
| GP rec         | 7%        | TVP    | 2%        |
| BSR av EW rec  | 5%        | TVP f  | 2%        |
|                |           | HA     | 0%        |

Source: own elaboration.

**Figure 3. The Giacomini-Rossi fluctuation test outcomes**



Source: own elaboration.

Figure 3 reports the Giacomini-Rossi fluctuation test outcomes (with  $\mu = 0.3$  corresponding to approximately 7.5-years periods, and with 5% significance level). In particular, the null hypothesis is that BSR rec and DMA forecasts accuracies are the same, and the alternative hypothesis is that BSR rec forecasts are worse than those generated by DMA model. The dotted line corresponds to the critical value of the statistic. The test statistics for different commodities are plotted in colours. For most commodities the test statistics are below the critical value in most of the time; therefore, the null hypothesis is not often rejected. However, between 2008 and 2016 in a reasonable number of cases the null hypothesis can be rejected, so it can be assumed that BSR rec forecasts are worse than DMA. In particular, the null hypothesis cannot be rejected in any period of time for 36 commodities: Australian



and South African coal, European natural gas, Japan liquefied natural gas, Colombo, Kolkata and Mombasa tea, groundnuts, fish meal, palm oil, soybeans, soybean oil, soybean meal, Thai 5% and 10% broken rice, US soft red winter and hard red winter wheat, US bananas, beef, chicken meat, European, US and world sugar, US import tobacco, Malaysian logs, Malaysian sawnwood, plywood, cotton (A index), phosphate rock, diammonium phosphate, triple superphosphate, urea, potassium chloride, iron ore, zinc, and silver.

## **5. Conclusions**

This study undertook the estimation of various econometric models addressing variable uncertainty across various commodities spot prices. Much attention was put on Bayesian Symbolic Regression (BSR), which, as a novel and yet not much explored tool, was anticipated to yield more precise forecasts compared to standard symbolic regression with genetic programming and other alternative models. However, BSR did not fully meet the anticipated expectations. Therefore, the main research question was negatively answered. However, this method still appears to possess great potential for economic and financial applications. On the other hand, Dynamic Model Averaging (DMA) was found as a highly effective method for forecasting commodities spot prices, which is in line with some previous studies (Drachal, 2018). Despite the lack of strong evidence to favour BSR in terms of forecast accuracy, this study still systematically compared numerous commonly used forecasting models across a wide basket of commodities. The obtained outcomes are also consistent with some other similar studies of BSR, but with different explanatory variables sets (Drachal 2023a; 2023b). In particular, it was found that rather prudent specification of the initial parameters for BSR would be quite sufficient. This can have a positive effect on the computational costs of the models applied in future studies. Moreover, in the case of the initial set of operators (functions), those representing quite common, simple, and standard transformations used in econometrics were found enough – with no great need to apply more complicated formulas. For example, moving average or lagging were found more important than transformations representing non-linear effects.

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### *Declaration of Generative AI and AI-assisted technologies in the writing process*

During the preparation of this work the author used ChatGPT in order to improve readability and language of few paragraphs and sentences in the work. After using this tool/service, the author reviewed and edited the content as needed and take full responsibility for the content of the publication.

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