The 3rd International Conference on Economics and Social Sciences
Innovative models to revive the global economy
October 15-16, 2020
Bucharest University of Economic Studies, Romania

Perfect Subgame Equilibrium in a Stackelberg Duopoly

Ciprian RUSESCU[1]

DOI: 10.2478/9788395815072-002

Abstract

Game theory is the perfect tool for modelling imperfect competition specific processes, manifested in relation to either the product quantity (Cournot or Stackelberg type), the product price (Bertrand type) or the quality. The equilibrium solution in output terms is highlighted in a Cournot scenario, whilst the price equilibrium solution can be revealed in a Bertrand situation. Despite the different strategy types on which the models are based, the common denominator is the fact that strategic choices are made simultaneously. The Stackelberg's model represents instead a perfect information sequential game (with firms advocating for quantity competition) and has both theoretical and practical applicability. In the simplest possible scenario, with two players moving in two stages, the leader will always choose a certain output level, while the follower will observe its decision and then establish its own action path accordingly. The main goal of this paper is to analyse a duopoly market with players adopting a Stackelberg behaviour. In any possible scenario, both firms are expected to survive and a stable equilibrium will manifest (the Subgame Perfect Equilibrium). The price will not react to the market demand curve slope variations and quantity and profit levels will be in an inverse dependence relationship with the aforementioned variable. The leader's chosen output and profit level will be higher than the output/profit of its follower.

Keywords: Stackelberg equilibrium, Stackelberg's model, Cournot's model, oligopoly, stability.

JEL classification: C72, D01, D43, L13

1. Introduction

As one fundamental representation of oligopoly games, the monopoly theory can be traced back two centuries ago when Antoine Augustine Cournot first put forward the mathematical model of duopoly competition (1838). Since then, Cournot's model became a starting point for the analysis of the oligopoly theory

[1] Bucharest University of Economic Studies, Bucharest, Romania, rusescuciprian18@stud.ase.ro.

presenting a duopoly scenario, with firms producing homogeneous products and choosing to compete in terms of quantities, while also simultaneously taking decisions regarding the production level.

After almost a century, another duopoly market model has been developed by Heinrich von Stackelberg, with players competing also in terms of quantities, but this time the decisions being taken consecutively. Also known as Stackelberg competition and being an imperfect competition model based on a non-cooperative game, this reviewed model is mostly an extension of Cournot's model.

The model was developed by Stackelberg in his 1934 book "Market Structure and Equilibrium" and represents a breaking point in the market structure analysis, especially in the duopoly scenario. Based on different starting assumptions and offering conclusions different from those of the Cournot and Bertrand's models, this new duopoly model is a sequential game with perfect information (unlike the Cournot's model, which is a simultaneous game).

As previously anticipated, the model has a real theoretical importance, but also practical importance. It can be used in industrial organizations to study the market structure determinants and other related aspects like market entry and entry pre-emption (Berry & Reiss, 2007; Mueller, 1986; Sutton, 2007). The Stackelberg's model is also an excellent tool for the analysis of the hierarchical structure scenario. Zhang & Zhang, 2009 used a Stackelberg game to model the problem of spectrum allocation in Cognitive Radio Networks. A Stackelberg game-based approach has also been used to model the problem of efficient bandwidth allocation in the cloud-based wireless networks, where desktop users watching the same live channel may be willing to share their live-streaming with the nearby mobile users (Nan et al., 2014). Stackelberg game models have been widely used in the security domain to illustrate the attacker-defender models (Pita et al., 2009 – Los Angeles International Airport protection against terrorists; Michael & Scheffer, 2011 – adversarial learning modelling in the setup when the adversary tries to manipulate the data miner's data to reduce the accuracy of the classifier.; Clempner & Poznyak, 2015, Trejo et al., 2015; etc.). To conclude, theoretical Stackelberg game models have been widely used to model different situations in various real market areas.

Further investigation of the influence of market demand curve slope on Stackelberg static equilibrium model highlights other aspects such as firm stability and demand curve slope impact on the perfect subgame equilibrium theory. The principles of the related mathematical model are also described below.

2. The Model

The background used is one with two firms, which sell homogeneous products, subject to the same demand and cost functions. One of them, the leader, has the right to make the first move, thanks to certain potential advantages such as market power, historical precedence, sophistication, size, reputation, innovation, information and so forth. Stackelberg assumes that this duopolist is sufficiently sophisticated to recognize that its competitor acts according to the Cournot assumption; this allows it to determine the reaction curve of its competitor and include it in its own profit function, acting as a monopolist in the attempt of maximizing its payoff. The other one, the follower, observes its strategy and decides about its own accordingly; its profit depends on the leader's chosen output level which is predetermined in its opinion, therefore will be considered an invariable information.

It is worthwhile mentioning that the leader's action is irreversible as it is aware ex ante that the follower observes its actions, establishing accordingly its own action path. The first mover advantage is undeniable, triggering the idea that the first player yields a higher payoff than the second player does.

An example of such leadership may be Microsoft's dominance in software markets. Although Microsoft can make decisions first, other smaller companies react to Microsoft's actions when making their own decisions. The actions of these followers, in turn, affect Microsoft. Another potential Stackelberg leadership scenario is highlighted in the aircraft industry: Airbus and Boeing competition (Waldman & Jensen, 2016).

Let's consider a general price function P(Q), which can be better expressed as $P(q_1 + q_2)$, giving the existing duopoly scenario, where q_1 and q_2 represent the leader/follower output level and Q the aggregate market demand:

$$P(Q) = P(q_1 + q_2)$$

We also assume that firm i has the cost structure $C_i(q_i), i = \overline{1,2}$.

To solve the model and find the Stackelberg equilibrium (perfect subgame equilibrium), we need to use backward induction, as in any sequential game. The leader anticipates the follower's best reaction, more precisely, how the latter will respond once it has observed its decision. The leader chooses the quantity q_1 which maximizes its payoff, to which the follower reacts by picking the expected quantity q_2. We should first determine the follower's best response function.

The profit function for the player 2 (the follower) will be:

$$\pi_2 = P(q_1 + q_2)q_2 - C_2(q_2)$$

The first order derivative expression can be seen below:

$$\frac{\delta \pi_2}{\delta q_2} = \frac{\delta P(q_1 + q_2)}{\delta q_2} q_2 + P(q_1 + q_2) - \frac{\delta C_2(q_2)}{\delta q_2}$$

whilst setting marginal profit expression to zero value opens the path for finding out the follower's best reply function.

We are looking forward now to the best reply function of the leader:

$$\pi_1 = P(q_1 + q_2(q_1))q_1 - C_1(q_1)$$

where $q_2(q_1)$ is the follower's quantity as a strictly dependent function of the leader's output, as we have previously agreed. Finally, the leader's marginal profit expression, who is leading to its best reply function, is described as follows:

$$\frac{\delta \pi_1}{\delta q_1} = \frac{\delta P(q_1 + q_2)}{\delta q_2} \frac{\delta q_2(q_1)}{\delta q_1} q_1 + \frac{\delta P(q_1 + q_2)}{\delta q_1} q_1 + P(q_1 + q_2(q_1)) - \frac{\delta C_1(q_1)}{\delta q_1}$$

Let's further consider the scenario of a downward slope of the linear demand curve, where the price dependence can be described as follows:

$$P(q_1 + q_2) = a - b(q_1 + q_2)$$

where a>0, b>0 and P represents the price paid by consumers for purchasing the required product quantity. The previously mentioned inverse demand function is getting close to the second product particular case of the mathematical formula used by Kresimir Zigic (2012) in his analysis, which presents a Stackelberg scenario using differentiated products (where b ∈(0,1) reflecting the degree of product differentiation or substitutability).

Adjusting the above mentioned formulas to the current hypothesis, the follower's profit function expression becomes:

$$\pi_2 = [a - b(q_1 + q_2)]q_2 - C_2(q_2)$$

Marginal profit expressions represent the starting point in the follower's reaction function, revealing: (see Appendix A):

$$q_2 = \frac{a - bq_1 - \frac{\delta C_2(q_2)}{\delta q_2}}{2b}$$

The spring of further determination of equilibrium values is represented by the leader's profit function:

$$\pi_1 = \big[a - b\big(q_1 + q_2(q_1)\big)\big]q_1 - C_1(q_1)$$

and the mathematical calculation (related calculations in Appendix A) leads to:

$$q_1^* = \frac{a + \frac{\delta C_2(q_2)}{\delta q_2} - 2\frac{\delta C_1(q_1)}{\delta q_1}}{2b} \qquad q_2^* = \frac{a - 3\frac{\delta C_2(q_2)}{\delta q_2} + 2\frac{\delta C_1(q_1)}{\delta q_1}}{4b}$$

where q_1^* represent the leader's best response to the follower's reaction, and q_2^* represents the follower's reaction function. That means the market demand level in the equilibrium scenario is:

$$Q^* = q_1^* + q_2^* = \frac{3a - \frac{\delta C_2(q_2)}{\delta q_2} - 2\frac{\delta C_1(q_1)}{\delta q_1}}{4b}$$

and further, the equilibrium price:

$$p^* = \frac{a + \frac{\delta C_2(q_2)}{\delta q_2} + 2\frac{\delta C_1(q_1)}{\delta q_1}}{4}$$

Referring now to the cost function, for the simplicity of calculation understanding, we can impose some mathematical restrictions:

$$\frac{\delta^2 C_i(q_i)}{\delta q_i\, \delta q_j} = 0; \ \frac{\delta C_i(q_i)}{\delta q_j} = 0; \ i,j = \overline{1,2}$$

and from all types of function dealing with, we pick the linear function $C_i(q_i) = c_i q_i$.

By also including this last hypothesis in our model, the Stackelberg perfect subgame equilibrium values become:

$$q_1^* = \frac{a-c}{2b} \qquad q_2^* = \frac{a-c}{4b} \qquad p^* = \frac{a+3c}{4}$$

$$\pi_1^* = \frac{(a-c)^2}{8b} \qquad \pi_2^* = \frac{(a-c)^2}{16b}$$

The results obtained lead to the following conclusions:

- $q_1^* > q_2^*$, meaning that the leader produces more; being more specific, the leader's output is twice as much the follower's;
- $p^* > c$, confirming for both players the possibility of making profits;
- $\pi_1^* > \pi_2^*$, the leader registers higher (double) profit, therefore highlighting a real advantage to move first. There are two main reasons leading to this: the leader knows that by increasing its output level, it will force the follower to reduce its own and this decision is irreversible (by undoing its action, we would reach the Cournot scenario).
- $Q^* > Q_{COURNOT} \rightarrow p^* < p_{COURNOT}$. The Stackelberg game leads to a more competitive equilibrium than the Cournot game.

We are now treating the b = 1 scenario - perfect substitute products. Therefore, p=a-q1-q2 highlighting the most simple possible form for price-output mathematical relation. Thus p+Q =a, meaning their sum remains constant, equalizing a parameter. The quantity offered by the leader will be $q_1 = \frac{a-c}{2}$ whilst the follower's response is $q_1 = \frac{a-c}{4}$. The price value suffered no modification $p = \frac{a+3c}{4}$ as it is not at all affected by the b parameter variation; looking further, we can observe that the leader/follower profit level became $\pi_1 = \frac{(a-c)^2}{8}$ respectively $\pi_2 = \frac{(a-c)^2}{16}$.

We further analyse the quantity/profit sensitivity to the b parameter value changes, in a perfect subgame equilibrium scenario (the price is not related to the slope, being constant at any a and c hypothetical value pairs.). All mathematical calculations representing the graphical analysis basis below are highlighted in the Appendix B, whilst in our simulation, we customise the a and c parameters, as follows: a = 100 EUR; c = 40 EUR. Using these assumptions, we start the gradual increase in the b parameter with a convenient ratio of 0.10, from the initial 0.1 value, up to the 3.0 final value.

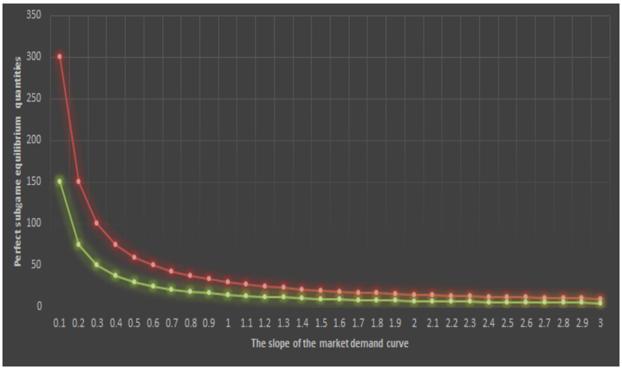

Figure 1. Perfect subgame equilibrium quantity evolution
Source: own processing

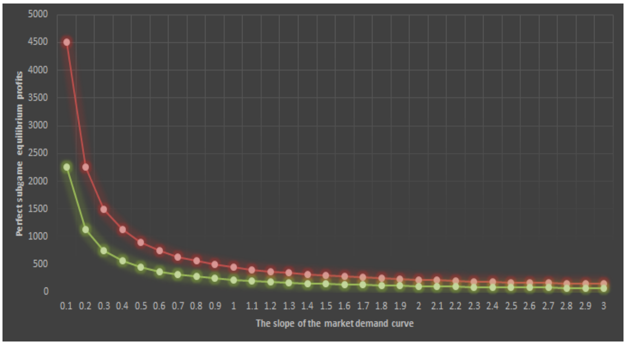

Figure 2. Perfect subgame equilibrium profit evolution
Source: own processing

Conclusions

Despite the fact that the b parameter value is continuously changing, it is easy to notice that the equilibrium price remains constant, due to the fact that its mathematical expression depends only on the a and c parameters. More precisely, regardless of the b value growth from 0.1 to 3, the equilibrium price still remains at the 55 EUR level.

As for the quantity triggering the equilibrium scenario, a downward trend is highlighted, starting with 5*(a-c) (the leader's case)/ 2.5*(a-c) (the follower's case). The explanation is also mathematical, deriving from the simple fact that $q_1^{*\prime} = -\frac{a-c}{2b^2}, q_2^{*\prime} = -\frac{a-c}{4b^2}$ are negative expressions, this kind of monotony being specific for the decreasing functions. Going further, $q_1^{*\prime\prime} = \frac{a-c}{b^3}, q_2^{*\prime\prime} = \frac{a-c}{2b^3}$, strictly positive second order derivatives explaining the convex type graph. By reference to the figures, the equilibrium quantity level registers a decreasing trend from its initial value of 300 kg (leader) / 150 kg (follower), down to zero value (not tangible, because y=0 represents an horizontal asymptote, as well as x=0 in fact).

In the profit equilibrium scenario, a downward trend can be noticed as well, starting from 1.25 (a-c)2 (leader) / 0.625 (a-c)2 (follower) down to zero, value which would also never be reached. Math principles offer one more time the key, as $\pi_1{}^{*\prime} = -\frac{(a-c)^2}{8b^2}, \pi_2{}^{*\prime} = -\frac{(a-c)^2}{16b^2}$, a strictly negative expression reflecting a decreasing function. From the same above mentioned reasons (second order positive derivatives), we meet a function convexity scenario. Given the previous hypothesis, a downward profit trend can be noticed, beginning with 4.500 EUR (leader) / 2.250 EUR (follower) down to a minimum profitability level (zero profit – not tangible, with y=0 also a horizontal asymptote).

3. Graphical Approach

In the next paragraphs, we shall try to explain the Stackelberg's model, making use of the graphical method, based on the duopolists' reaction functions. First of all, we will deduce the general expression of the leader's isoprofit curve, and looking forward, its competitor's best response:

$\pi^1(q_1, q_2) = [a - b(q_1 + q_2) - c]q_1$, then $\bar{\pi} = [a - b(q_1 + q_2) - c]q_1 = aq_1 - bq_1^2 - bq_1q_2 - cq_1$

$$\rightarrow bq_1q_2 = (a-c)q_1 - bq_1^2 - \bar{\pi} \rightarrow q_2 = \frac{a-c}{b} - q_1 - \frac{\bar{\pi}}{bq_1}$$

Each isoprofit curve highlights a constant level of profit that could be obtained by one player at different output levels chosen by it and its competitor. The follower's first order derivative expression offers very important information regarding the isoprofit curve trend (ascending/descending), whilst the second order derivative highlights the concavity related to the axes:

$$\frac{dq_2}{dq_1} = -1 + \frac{\bar{\pi}}{bq_1^2} \rightarrow \frac{d^2q_2}{dq_1^2} = -\frac{2\bar{\pi}}{bq_1^3} < 0$$

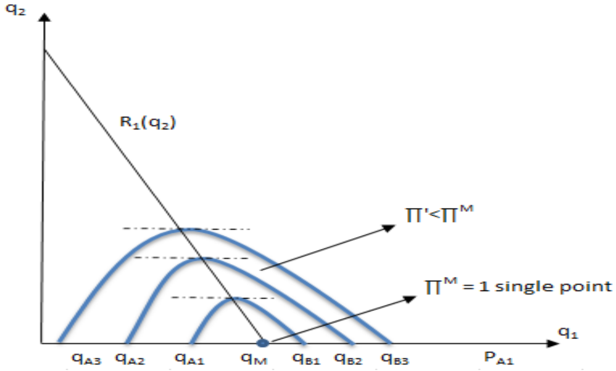

Figure 3. Leader's isoprofit and best reply functions
Source: own processing

Firm 1 (the leader) will always choose its best response, reflected on the isoprofit curve that corresponds to the maximum profit, at a q_2 given level.

The point of intersection of the reaction function with the isoprofit curves has the mathematical zero slope property (Machado, 2008)

$$R_1(q_2) = argmax\, \pi^1(q_1, q_2) \;\rightarrow\; \pi_1^1(R_1(q_2), q_2) = 0.$$

Besides, we already know that $\pi^1(q_1, q_2) = \bar{\pi} \rightarrow \pi_1^1 dq_1 + \pi_2^1 dq_2 = 0 \rightarrow \frac{dq_2}{dq_1} = -\frac{\pi_1^1}{\pi_2^1}$, then the resulting derivative $\frac{dq_2}{dq_1}$ should be nil in leader's best response scenario $q_1 = R_1(q_2)$.

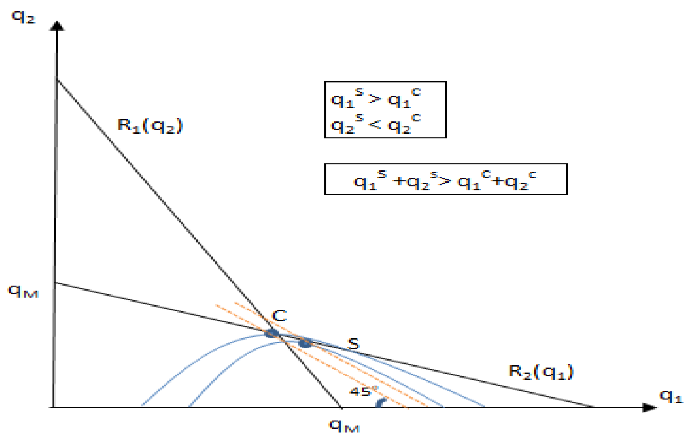

Figure 4. Stackelberg equilibrium vs. Cournot equilibrium
Source: own processing

The leader's optimal behaviour is reached in the tangency point S of its isoprofit curve with the reaction curve of the follower (firm 2), whilst C would be the Cournot equilibrium, where the reaction curves cross and where dq2/dq1=0 (as previously mentioned). All three mentioned relations can be easily proved either by comparing the specific equilibrium values (see below) of Stackelberg & Cournot's models or by a simple figure analysis.

$$q_1^S = \frac{a-c}{2b} = \frac{3}{2}\frac{a-c}{3b} = \frac{3}{2}\, q_1^C > q_1^C \qquad q_2^S = \frac{a-c}{4b} = \frac{3}{4}\frac{a-c}{3b} = \frac{3}{4}\, q_2^C < q_2^C$$

$$Q^S = q_1^S + q_1^S = \frac{3(a-c)}{4b} > \frac{2(a-c)}{3b} = q_1^C + q_1^C \; = Q^C$$

$$p^S = \frac{a+3c}{4} < \frac{a+2c}{3} = p^C \overset{a>c}{\longleftarrow} \frac{a+3c}{4} < \frac{a+2c}{3} \rightarrow 3a + 9c < 4a + 8c \rightarrow c < a\ (A)$$

$$\pi_1^S = \frac{(a-c)^2}{8b} = \frac{9}{8}\frac{(a-c)^2}{9b} = \frac{9}{8}\pi_1^C > \pi_1^C \quad \pi_2^S = \frac{(a-c)^2}{16b} = \frac{9}{16}\frac{(a-c)^2}{9b} = \frac{9}{16}\pi_2^C < \pi_2^C$$

Conclusions: In the symmetric firms scenario (costs are matching), the Stackelberg solution is superior to the Cournot solution (higher aggregate output, lower price, higher aggregate profits). From the other point of view, the first firm profit level should not be lower than in Cournot scenario because the leader could have always obtained the Cournot profit levels by simply choosing the Cournot quantity q_1^C, to which its competitor would have replied with its Cournot quantity $q_2^C = R_2(q_1^C)$, as the follower's reaction curve in Stackelberg is the same as in Cournot.

References

[1] Berry, S., & Reiss, P. (2007). Empirical models of entry and market structure. In: *Handbook of Industrial Organization*, 3, pp. 1845-1886.

[2] Clempner, J. B., & Poznyak, A. S. (2015). *Stackelberg Security Games: Computing The Shortest-Path Equilibrium. Expert Systems with Applications*, 42, pp. 3967-3979.

[3] Machado, M. (2008). *Economia Industrial*, Cap. 3.2 - Competencia (secuencial) en cantidades Modelo de Stackelberg. Retrieved from https://www.academia.edu/16477977/1_1.Monopolio_modelo_basico.

[4] Michael, B., & Scheffer, T. (2011). Stackelberg Games for Adversarial Prediction Problems. In *Proceedings of the 17th ACM SIGKDD International Conference on Knowledge Discovery and Data Mining*, pp. 547-555. San Diego, California, USA: ACM. doi:10.1145/2020408.2020495.

[5] Mueller, D.C. (1986). *Profits in the Long Run*. Cambridge University Press.

[6] Nan, G. et al. (2014). Stackelberg Game for Bandwidth Allocation in Cloud-based Wireless Live-streaming Social Networks. *IEEE Systems Journal*, 8, pp. 256-267.

[7] Pita, J., Jain, M., Ordonez, F., Portway, C., Tambe, M., Western, C., Paruchuri, P., Kraus, S. (2009). Using Game Theory for Los Angeles Airport Security. *AI MAGAZINE*, 30, pp. 43-57. doi: http://dx.doi.org/10.1609/aimag.v30i1.2173.

[8] Sutton, J. (2007). Market structure: theory and evidence. In *Handbook of Industrial Organization*, 3, pp. 2301-2368.

[9] Trejo, K. K., Clempner, J. B., & Poznyak, A. S. (2015). A Stackelberg security game with random strategies based on the extraproximal theoretic approach. *Engineering Applications of Artificial Intelligence*, 37, pp. 145-153. doi:10.1016/j.engappai.2014.09.002.

[10] Von Stackelberg, H. (1934). *Marktform und Gleichgewicht* [Market Structure and Equilibrium - translated by Damien Bazin, Lynn Urch and Rowlnd Hill].

[11] Waldman, D. E., Jensen, E. J. (2016). *Industrial organisation. Theory and practice*, Forth Edition.

[12] Zhang, J., & Zhang, Q. (2009). Stackelberg Game for Utility-Based Cooperative Cognitive Radio Networks. In U. New Orleans, LA (Ed.), *Proceedings of the Tenth ACM International Symposium on Mobile Ad Hoc Networking and Computing*, pp. 23-32. ACM. doi:10.1145/1530748.1530753.

[13] Žigić, K. (2012). Stackelberg leadership with product differentiation and endogenous entry: some comparative static and limiting results. *Journal of Economics*, 99(3), pp. 251-266.

Appendix A

$$\frac{\delta\pi_2}{\delta q_2} = 0 \rightarrow \frac{\delta[a - b(q_1 + q_2)]}{\delta q_2} q_2 + a - b(q_1 + q_2) - \frac{\delta C_2(q_2)}{\delta q_2}$$

$$= -bq_2 + a - b(q_1 + q_2) - \frac{\delta C_2(q_2)}{\delta q_2} = 0$$

$$-2bq_2 = \frac{\delta C_2(q_2)}{\delta q_2} - a + bq_1 \;\rightarrow\; q_2 = \frac{a - bq_1 - \frac{\delta C_2(q_2)}{\delta q_2}}{2b}$$

$$\pi_1 = \left[a - b\left(q_1 + q_2(q_1)\right)\right] q_1 - C_1(q_1)$$

$$= \left(a - b\left(q_1 + \frac{a - bq_1 - \frac{\delta C_2(q_2)}{\delta q_2}}{2b} \right) \right) q_1 - C_1(q_1)$$

$$= \left(a - bq_1 - \frac{a - bq_1 - \frac{\delta C_2(q_2)}{\delta q_2}}{2} \right) q_1 - C_1(q_1) = \frac{a - bq_1 + \frac{\delta C_2(q_2)}{\delta q_2}}{2} q_1 - C_1(q_1)$$

$$\frac{\delta\pi_1}{\delta q_1} = 0 \rightarrow \frac{a - bq_1 + \frac{\delta C_2(q_2)}{\delta q_2}}{2} - \frac{bq_1}{2} - \frac{\delta C_1(q_1)}{\delta q_1} = \frac{a - 2bq_1 + \frac{\delta C_2(q_2)}{\delta q_2}}{2} - \frac{\delta C_1(q_1)}{\delta q_1} = 0$$

$$-bq_1 + \frac{a + \frac{\delta C_2(q_2)}{\delta q_2}}{2} - \frac{\delta C_1(q_1)}{\delta q_1} = 0 \rightarrow q_1^* = \frac{a + \frac{\delta C_2(q_2)}{\delta q_2} - 2\frac{\delta C_1(q_1)}{\delta q_1}}{2b}$$

$$q_2^* = \frac{a - b\frac{a + \frac{\delta C_2(q_2)}{\delta q_2} - 2\frac{\delta C_1(q_1)}{\delta q_1}}{2b} - \frac{\delta C_2(q_2)}{\delta q_2}}{2b} \rightarrow q_2^* = \frac{a - 3\frac{\delta C_2(q_2)}{\delta q_2} + 2\frac{\delta C_1(q_1)}{\delta q_1}}{4b}$$

$$q^* = q_1^* + q_2^* = \frac{a + \frac{\delta C_2(q_2)}{\delta q_2} - 2\frac{\delta C_1(q_1)}{\delta q_1}}{2b} + \frac{a - 3\frac{\delta C_2(q_2)}{\delta q_2} + 2\frac{\delta C_1(q_1)}{\delta q_1}}{4b}$$

$$= \frac{3a - \frac{\delta C_2(q_2)}{\delta q_2} - 2\frac{\delta C_1(q_1)}{\delta q_1}}{4b}$$

$$p^* = a - b(q_1^* + q_2^*) = a - b\frac{3a - \frac{\delta C_2(q_2)}{\delta q_2} - 2\frac{\delta C_1(q_1)}{\delta q_1}}{4b} \rightarrow$$

$$p^* = \frac{a + \frac{\delta C_2(q_2)}{\delta q_2} + 2\frac{\delta C_1(q_1)}{\delta q_1}}{4}$$

$$C_i(q_i) = cq_i \rightarrow \frac{\delta C_i(q_i)}{\delta q_i} = c_i(q_i) = c, (\forall)\; i = \overline{1,2}$$

$$q_1^* = \frac{a - c}{2b} \qquad q_2^* = \frac{a - c}{4b} \qquad p^* = \frac{a + 3c}{4}$$

$$\pi_1^* = (p^* - c)q_1^* = \left(\frac{a+3c}{4} - c\right)\frac{a-c}{2b} = \frac{(a-c)^2}{8b}$$

$$\pi_2^* = (p^* - c)q_2^* = \left(\frac{a+3c}{4} - c\right)\frac{a-c}{4b} = \frac{(a-c)^2}{16b}$$

$$q_1^* = 2q_2^* \qquad \pi_1^* = 2\pi_2^*$$

Appendix B

Table 1. Simulation of price, quantity and profit evolution

b	p	q_1	q_2	π_1	π_2
0.1	0.25*a+0.75*c	5.000000*(a-c)	2.500000*(a-c)	1.250000*(a-c)^2	0.625000*(a-c)^2
0.2	0.25*a+0.75*c	2.500000*(a-c)	1.250000*(a-c)	0.625000*(a-c)^2	0.312500*(a-c)^2
0.3	0.25*a+0.75*c	1.666667*(a-c)	0.833333*(a-c)	0.416667*(a-c)^2	0.208333*(a-c)^2
0.4	0.25*a+0.75*c	1.250000*(a-c)	0.625000*(a-c)	0.312500*(a-c)^2	0.156250*(a-c)^2
0.5	0.25*a+0.75*c	1.000000*(a-c)	0.500000*(a-c)	0.250000*(a-c)^2	0.125000*(a-c)^2
0.6	0.25*a+0.75*c	0.833333*(a-c)	0.416667*(a-c)	0.208333*(a-c)^2	0.104167*(a-c)^2
0.7	0.25*a+0.75*c	0.714286*(a-c)	0.357143*(a-c)	0.178571*(a-c)^2	0.089286*(a-c)^2
0.8	0.25*a+0.75*c	0.625000*(a-c)	0.312500*(a-c)	0.156250*(a-c)^2	0.078125*(a-c)^2
0.9	0.25*a+0.75*c	0.555556*(a-c)	0.277778*(a-c)	0.138889*(a-c)^2	0.069444*(a-c)^2
1.0	0.25*a+0.75*c	0.500000*(a-c)	0.250000*(a-c)	0.125000*(a-c)^2	0.062500*(a-c)^2
1.1	0.25*a+0.75*c	0.454545*(a-c)	0.227273*(a-c)	0.113636*(a-c)^2	0.056818*(a-c)^2
1.2	0.25*a+0.75*c	0.416667*(a-c)	0.208333*(a-c)	0.104167*(a-c)^2	0.052083*(a-c)^2
1.3	0.25*a+0.75*c	0.384615*(a-c)	0.192308*(a-c)	0.096154*(a-c)^2	0.048077*(a-c)^2
1.4	0.25*a+0.75*c	0.357143*(a-c)	0.178571*(a-c)	0.089286*(a-c)^2	0.044643*(a-c)^2
1.5	0.25*a+0.75*c	0.333333*(a-c)	0.166667*(a-c)	0.083333*(a-c)^2	0.041667*(a-c)^2
1.6	0.25*a+0.75*c	0.312500*(a-c)	0.156250*(a-c)	0.078125*(a-c)^2	0.039063 (a-c)^2
1.7	0.25*a+0.75*c	0.294118*(a-c)	0.147059*(a-c)	0.073529*(a-c)^2	0.036765*(a-c)^2
1.8	0.25*a+0.75*c	0.277778*(a-c)	0.138889*(a-c)	0.069444*(a-c)^2	0.034722*(a-c)^2
1.9	0.25*a+0.75*c	0.263158*(a-c)	0.131579*(a-c)	0.065789*(a-c)^2	0.032895*(a-c)^2
2.0	0.25*a+0.75*c	0.250000*(a-c)	0.125000*(a-c)	0.062500*(a-c)^2	0.031250*(a-c)^2
2.1	0.25*a+0.75*c	0.238095*(a-c)	0.119048*(a-c)	0.059524*(a-c)^2	0.029762*(a-c)^2
2.2	0.25*a+0.75*c	0.227273*(a-c)	0.113636*(a-c)	0.056818*(a-c)^2	0.028409*(a-c)^2
2.3	0.25*a+0.75*c	0.217391*(a-c)	0.108696*(a-c)	0.054348*(a-c)^2	0.027174*(a-c)^2
2.4	0.25*a+0.75*c	0.208333*(a-c)	0.104167*(a-c)	0.052083*(a-c)^2	0.026042*(a-c)^2
2.5	0.25*a+0.75*c	0.200000*(a-c)	0.100000*(a-c)	0.050000*(a-c)^2	0.025000*(a-c)^2
2.6	0.25*a+0.75*c	0.192308*(a-c)	0.096154*(a-c)	0.048077*(a-c)^2	0.024038*(a-c)^2
2.7	0.25*a+0.75*c	0.185185*(a-c)	0.092593*(a-c)	0.046296*(a-c)^2	0.023148*(a-c)^2
2.8	0.25*a+0.75*c	0.178571*(a-c)	0.089286*(a-c)	0.044643*(a-c)^2	0.022321*(a-c)^2
2.9	0.25*a+0.75*c	0.172414*(a-c)	0.086207*(a-c)	0.043103*(a-c)^2	0.021552*(a-c)^2
3.0	0.25*a+0.75*c	0.166667*(a-c)	0.083333*(a-c)	0.041667*(a-c)^2	0.020833*(a-c)^2

Source: own processing