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Bertrand Competition Under Incomplete Information

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Abstract

The complexity of economic games has determined the application of models from game theory in the process of making optimal price decisions. Thus, price competition is perceived as a game in which microeconomic entities have different market powers and market performance depending on the informational advantage and implicitly on the opportunities generated by its capitalization. The various models developed having as reference the classical model Bertrand (1883) provide useful insights into the behaviour of firms in price competition and the operation of firms in market. Since the informational structure is one of the most important dimensions of the decision-making process, Bertrand-type competition analysis through Bayesian games under incomplete information is gaining attention in current research. The peculiarity of these games refers to the association of subjective probabilities with the parameters of the game, each player aiming to extract the informational rent of the opponents. So, the results of Bayesian games are influenced by the private information held by at least one player, the information gap between players making strategic selection difficult. This article aims to provide an overview of the Bertrand competition in incomplete information (unknown demand and costs). Taking into account the scenario in which close, but not perfect substitutes exist for the differentiated product with hypothetical data, we determine the equilibrium price and the equilibrium profit levels followed by a simulation to outline the variation of gains according to the subjective distributions on the choice of strategies by opponents.

Keywords: Bertrand model, Bayesian game, incomplete information, game theory, industrial organization.

JEL Classification: C72, D43, L13.

1. Introduction

The informational structure that subsists in an economic game whose objective is the choice of the price estimation strategy acquires specific connotations when

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not only the theoretical contexts are analysed, but also the particular ones. Depending on the preference over risk, the distorted perception of a circumstance by decision makers is present especially in the case of probabilities that register values close to the extremes. The continuous concern of firms for the economic value created can explain a wide range of behavioural anomalies. Both the informational advantage and the performance of microeconomic entities impact the informational structure regarding the behavioural side of the decision factor. The information asymmetry implies the existence of small deviations from the optimal behaviour of firms that, most of the times, have significant economic implications, being considered the source of the power imbalance during economic transactions and economic inefficiency respectively.

Game theory is an important branch of microeconomics that provides a modelling tool for decision-making in competitive situations. These situations are influenced by several rational factors, which act independently in the choice of strategic decisions, but they are dependent on the results comprising the set of all decisions and are ultimately outlining a complex set of interdependencies. This situation is formalized in the mathematical concept of the game.

As the price estimation strategy shows how economic agents interact in different types of markets while allocating their limited resources, the optimal decision can be defined only within the limits of a mathematical model. Given the wide applicability of the game theory in shaping price strategies, the idea of playing in incomplete information best outlines the economic reality. Economic agents have certain assumptions about the payoff function of their opponents. Certain probabilities are associated to these assumptions in order to highlight the behaviour of the players, the reason behind their decisions, the strategic landmarks, and the untapped opportunities that compensate the constraints of the game.

This paper is organised as follows. The first part introduces a review of the literature on the Bertrand model, including the most important references to the research objectives. We continue with the description of the methodology and the variable used in the model, followed by a simulation. The last part is dedicated to the interpretation of the results and drawing conclusions regarding the Bertrand-type competition in incomplete information along with identifying future research directions.

2. Problem Statement

The models applied for the description of the Bertrand-type competition are based on specific concepts and results of games with informational asymmetry. These models can be placed into the Bayesian universe mostly by considering the complexity of economic games, the formation of beliefs by players regarding the private information held by opponents and the adjustment of beliefs during the development of economic relations.

Harsanyi (1967) defined Bayesian games assuming that uncertainty in games can be modelled equivalently as uncertainty related to the payoff functions. In the Bayesian games formulated by Harsanyi, the different information of the players is described by a collection of random variables, called types of players, representing the private information of a player. The actual value of each player's type has been omitted from the model. However, a precise probabilistic description of what each player type would think about the other player's types is included. This approach provides a general framework for the entire economic model applied.

Singh and Vives (1984) identified three important aspects of oligopolistic competition, which have been subsequently analysed in various studies. Assuming that the goods are substitutable, they found that the players' profits are significantly higher in the case of Cournot-type competition than in the case of Bertrand-type competition, while consumer and social welfare are higher in the case of price competition. Firms that choose Cournot-type competition represent a dominant strategy equilibrium in the two-stage game in which firms choose between the two types of competition in the first stage and then compete accordingly in the second stage.

Häckner (2000) emphasized the efficiency of the two competition models considering the asymmetric demand as the difference in quality between goods. Cournot-type competition always produces higher prices and lower welfare compared to Bertrand-type competition. Profits are higher in Cournot-type competition for substitutable good’s while Bertrand-type competition is more profitable for complementary goods. Finally, it is a dominant strategy for firms to choose quantity as a strategic variable when goods are substitutable, and prices when they are complementary.

Yue et al. (2006) presented a model of profit maximization in order to obtain optimal strategies in conditions of information asymmetry. The model follows a Bertrand-type game with players having private information. Buyers have a choice of two complementary goods offered by two players in the form of a mix package. The quantities offered by each player depend on both their own pricing strategy and that of the opponent. The authors compared the results obtained in conditions of asymmetric information, information sharing and strategic alliance, and they concluded that the exchange of information can create both advantages and disadvantages for players. The main advantage of information sharing is an increase in the accuracy of the planning process. In the case of the strategic alliance and information exchange, players would set prices lower than the optimal prices. Thus, players behave as a single microeconomic entity, sharing their profit equally.

Abbink and Brandts (2008) analysed the pricing process under the assumption of increasing marginal costs. They pointed out that, in the long run, the model involves prices converging towards the Walrasian result. In the case of the Bertrand model with strictly convex costs, firms can record a profit despite decreasing returns to scale. These results were previously obtained by Bulow et al. (1985) and Dastidar (1995), and subsequently by Argenton and Müller (2012).

Ferreira and Pinto (2011) found that the results of the Bertrand model are significantly influenced by the presence of differentiated goods or by the asymmetry of costs. In their paper, they analysed the Bertrand-type competition with differentiated goods, assuming that each player has two different technologies at disposal and he chooses one of them based on a certain distribution of probability. Based on these hypotheses, they determined the Bayesian-Bertrand Nash equilibrium. They also showed that there is a direct relationship between the expected price of each good and the expected costs, as the result of the effect of the expected cost of the rival dominated by the effect of its own expected cost.

Fatas et al. (2014) considered the Bertrand model in which firms take into account a discrete increment to adjust their prices. Through an empirical analysis, they referred to the model of limited rationality, a concept defined by Simon (1955). In the case of a sequential game, this model contributes to the adjustment of the firms' behaviour only in the first stages of the game. These results are in line with those obtained by Dufwenberg and Gneezy (2000).

Blavatskyy (2018) extended the classic Bertrand model by making various assumptions regarding the behaviour of consumers and firms. The initial set of hypotheses includes the following: the decision on whether to buy a good from a firm belongs to the consumer, a firm cannot increase its sales by increasing the price, and the relative market shares of firms do not change when all firms modify the value of the prices by the same amount or the same percentage in the set of hypotheses considered. According to these assumptions, the equilibrium price may be higher than the marginal cost of these firms. However, an equilibrium price does not necessarily converge to marginal cost when there are infinite firms in the market. The market price is lower when consumers are more sensitive to price changes and it converges to the marginal cost of firms as in the classic model Bertrand (1883).

Rusescu and Roman (2020) performed a comparison between Cournot, Bertrand and Stackelberg competition for games in complete information, with simultaneous and sequential decisions. They determine the optimal solutions for the case of products differentiation.

3. Methodology and Model

Since the competitors' response to the decisions given by the price evolution is the key element of the firms' analysis, it is important to know the factors contributing to pricing strategy. Degree of asymmetric information, quality of information, market conditions are among the factors that contribute to estimating a sustainable pricing strategy. Bayesian games allow these factors to be seen as a state of nature, respectively as random variables. The classical Bertrand model involves two companies that chose the prices for two substitutable goods.

In our case, we consider the Bayesian game that involves specifying four elements:

i. Action space: The prices set by the two players: $P_i = [0, \infty),\ i = 1, 2$.
ii. Types of players: These are given by the costs they record. Player 1 is of one type $T_1 = \{c\}$, and player 2 of two types: with low or high marginal cost $T_2 = \{c_M, c_m\}$.
iii. Probability space: the set of probabilities with which player 1 makes assumptions about the opponent's costs. It assumes with the probability θ that

the opponent has high costs (c_M) and with the probability (1-θ) that he has low costs (c_m) .

iv. Payoffs space - Profits of the two firms:

$\pi_i(p_i, p_j) = (p_i - c_i)(x_i - y_i p_i + z_i p_j)$, *i,j = 1, 2, i ≠j*

Thus, Bayesian equilibrium of the game:

$$P = P_1 \times P_2 = [0, \infty) \quad (1)$$

where:

$$\begin{cases} p_1 \text{is the price chosen by the first player,} p_1 \in P_1 = (0, \infty) \\ p_2 = \{p_2^m, p_2^M\} \text{ (depending on costs, } c_m \text{ or } c_M) \end{cases} \quad (2)$$

To determine the equilibrium prices, we assume $q_i, i = \overline{1, 2}$ demand functions, which are linear, differentiable and continuous for both firms:

$$\begin{cases} q_1 = x_1 - y_1 p_1 + z_1 p_2 \\ q_2 = x_2 - y_2 p_2 + z_2 p_1 \end{cases} \quad (3)$$

where:

- $x = x_1 + x_2$ is the maximum quantity demanded on the market;
- y_i expresses the marginal demand of the good *i* in relation to its price. We have normal goods only if $y_i > 0$;
- z_i is the cross-marginal demand in relation to the other firm price.

The average cost faced by firm 1 (hereafter F_1) is known by both competitors and firm 2 (hereafter F_2) can have two types of cost. Therefore, F_2 has private information regarding its costs, company F_1 can only make assumptions about the type of costs faced by its opponent.

$$\begin{cases} C_1(q_1) = c * q_1 \\ C_2(q_2) = \begin{cases} c_m q_2 \\ c_M q_2 \end{cases}, 0 \leq c_m \leq c_M \end{cases} \quad (4)$$

4. Model solutions

Having the hypotheses highlighted, we determine the equilibrium prices in a Bayesian-Bertrand game with two firms that compete by simultaneously setting prices.

Depending on the registered cost, we express the profit of F_2 as follows:

$\pi_2^M = (p_2^M - c_2^M)(x_2 - y_2 p_2^M + z_2 p_1)$, if F_2 faces high costs

or (5)

$\pi_2^m = (p_2^m - c_2^m)(x_2 - y_2 p_2^m + z_2 p_1)$, if F_2 faces low costs

To write F_1's profit, we must specify the lottery he faces:

$$\text{U} : \begin{pmatrix} (p_1 - c)(x_1 - y_1 p_1 + z_1 p_2^M) & (p_1 - c)(x_1 - y_1 p_1 + z_1 p_2^m) \\ \theta & 1 - \theta \end{pmatrix} \quad (6)$$

The expected win (expected utility) for F_1 that is in information asymmetry is calculated based on the lottery defined in equation 6 as an average win between the profit obtained if F_2 establishes high prices and the profit obtained if it establishes low prices.

Its expected win is the previous lottery average, respectively:

$$\pi_1 = E(U) = \theta * (p_1 - c)(x_1 - y_1 p_1 + z_1 p_2^M) + (1-\theta) * (p_1 - c)(x_1 - y_1 p_1 + z_1 p_2^m \qquad (7)$$

Given the fact that firms make decisions simultaneously, their reaction functions form a system from which we determine the equilibrium of the game. The two microeconomic entities, being rational, aim at maximizing profit. Therefore, the reaction functions are obtained as a solution of the optimization program for each company.

Under the given conditions, two optimization programs are required for F_2 as follows:

i. if F_2 faces high costs

$$\max_{p_2^M} \pi_2^M = (p_2^M - c_2^M)(x_2 - y_2 p_2^M + z_2 p_1) \qquad (8)$$

The reaction function of F_2 is determined by the first order condition:

$$\frac{\partial \pi_2^M}{\partial p_2^M} = 0 \Rightarrow p_2^M = \frac{x_2 + c_2^M y_2}{2y_2} + \frac{z_2 p_1}{2y_2} \text{ reaction function of firm } F_2 \qquad (9)$$

ii. if F_2 faces low costs

$$\max_{p_2^m} \pi_2^m = (p_2^m - c_2^m)(x_2 - y_2 p_2^m + z_2 p_1) \qquad (10)$$

with the following reaction function:

$$\frac{\partial \pi_2^m}{p_2^m} = 0 \Rightarrow p_2^m = \frac{x_2 + c_2^m y_2}{2y_2} + \frac{z_2 p_1}{2y_2} \qquad (11)$$

The calculation of the reaction function is shown in Appendix 1.

The optimization program corresponding to firm F_1 involves maximizing profit when the type of firm F_2 is unknown, but assumptions are made regarding this.

$$\max_{p_1} \pi_1 = \theta * (p_1 - c)(x_1 - y_1 p_1 + z_1 p_2^M) + (1-\theta) * (p_1 - c)(x_1 - y_1 p_1 + z_1 p_2^m) \qquad (12)$$

The optimization is done in relation to the price p_1, and the first order condition is:

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \frac{x_1 + c y_1}{2y_1} + \frac{z_1[\theta p_2^M + (1-\theta) p_2^m]}{2y_1} \text{ reaction function of firm } F_1 \qquad (13)$$

The system formed by the reaction functions of the two firms consists of three equations and three unknowns:

$$\begin{cases} p_2^M = \frac{x_2 + c_2^M y_2}{2y_2} + \frac{z_2 p_1}{2y_2} \\ p_2^m = \frac{x_2 + c_2^m y_2}{2y_2} + \frac{z_2 p_1}{2y_2} \\ p_1 = \frac{x_1 + c y_1}{2y_1} + \frac{z_1[\theta p_2^M + (1-\theta) p_2^m]}{2y_1} \end{cases} \qquad (14)$$

The solution of the system is found in Appendix 2. The equilibrium price established by firm F_1 is determined by replacing the reaction functions of firm F_2 in the reaction function of firm F_1:

$$p_1^* = \frac{2x_1y_1+2cy_1y_2+z_1y_2[\theta c_2^M+(1-\theta)c_2^m]+z_1x_2}{4y_1y_2-z_1z_2} \quad (15)$$

The equilibrium prices for firm F_2 are obtained by replacing the equilibrium price of firm F_1 in the reaction functions of firm F_2 as follows:

$$\begin{cases} p_2^{M*} = \frac{4x_2y_1y_2+(4y_1y_2^2-y_2z_1z_2)c_2^M+2x_1y_1z_2+2cy_1y_2z_2}{2y_2(4y_1y_2-z_1z_2)} + \frac{z_1z_2[\theta c_2^M+(1-\theta)c_2^m]}{2(4y_1y_2-z_1z_2)} \\ p_2^{m*} = \frac{4x_2y_1y_2+(4y_1y_2^2-y_2z_1z_2)c_2^m+2x_1y_1z_2+2cy_1y_2z_2}{2y_2(4y_1y_2-z_1z_2)} + \frac{z_1z_2[\theta c_2^M+(1-\theta)c_2^m]}{2(4y_1y_2-z_1z_2)} \end{cases} \quad (16)$$

The obtained solutions are considered to be optimal if it satisfies the conditions of non-negativity: $p_1{}^*, p_2^{M^*}, p_2^{m^*} \geq 0$. So, $4y_1y_2 \neq z_1z_2$ and $4y_1y_2 > z_1z_2$.

For the considered case, π_1^*, π_2^* represent the profit functions:

$$\begin{cases} \pi_2^{M*} = (p_2^{M*} - c_2^M)(x_2 - y_2p_2^{M*} + z_2p_1^*) \\ \pi_2^{m*} = (p_2^{m*} - c_2^m)(x_2 - y_2p_2^{m*} + z_2p_1^*) \\ \pi_1^* = \theta * (p_1^* - c)(x_1 - y_1p_1^* + z_1p_2^{M*}) + (1-\theta) * (p_1^* - c)(x_1 - y_1p_1^* + z_1p_2^{m*}) \end{cases} \quad (17)$$

5. Findings

Following the methodology, we analyse the evolution of prices and profits taking into account the multitude of probabilities with which F_1 makes assumptions regarding the costs of F_2. Let us consider the scenario in which close, but not perfect substitutes exist for the differentiated product and an asymmetric demand.

Equilibrium prices and profits are determined on the basis of the following hypothetical data:

x_1	x_2	y_1	y_2	z_1	z_2	c^M	c_m	c	θ
108	96	1	1	0.8	0.9	20	10	16	[0.01;0.99]

We consider the case variation of probability θ in the [0.01, 0.99] interval with an increment step of 0.01. We show what happens with prices and profits depending on the size of the cost asymmetry. The price evolution is depicted in Figure 1.

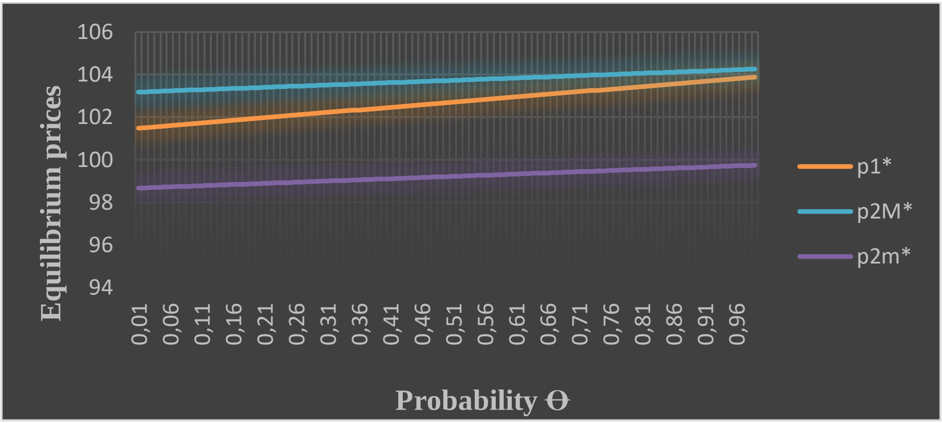

Figure 1. Price evolution
Source: Own processing.

By analysing the results, we notice that, regardless of the assumptions of F_1, the prices set by it approach the level of prices set by F_2 in case it faces high costs. This denotes the F_1's risk aversion. If F_1 assumes with a probability in the range [0.01, 0.54] that F_2 has high costs, the prices set by it are between 1% and 1.6% lower than the opponent's prices. When it assumes with a probability that tends to 1 that F_2 has high costs, the price levels set by the two firms are very close. As the cost difference between firms diminishes, the market price becomes more anti-competitive. Regardless of the level of assumptions about the type of opponent's costs, the prices of company F_1 are at least 2.86% higher than if F_2 faces low costs. Even if the firms compete on the same market, they can present homogeneous but not identical cost conditions that affect the competitive character.

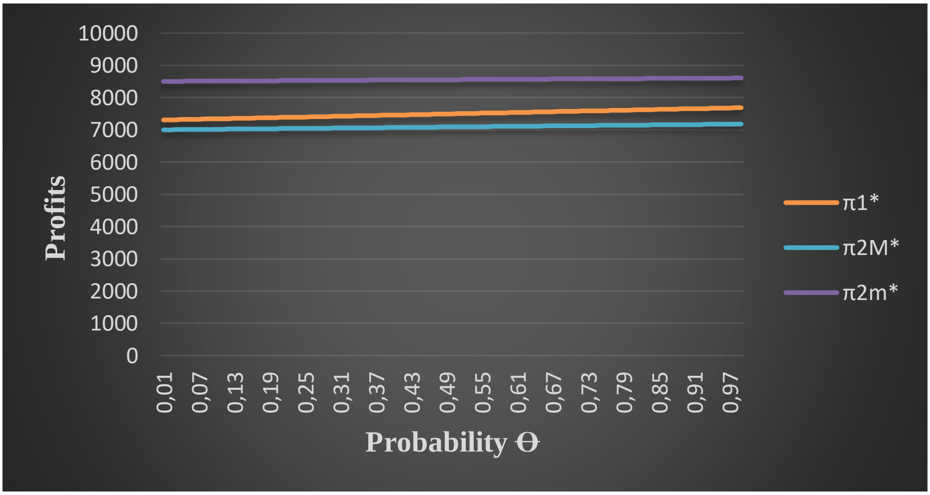

Figure 2. Profit evolution
Source: Own processing.

Regarding the evolution of profits represented in figure 2, F_1 can register higher profits only when F_2 has high costs. In this case, the gap between the profits of the two becomes larger as F_1 believes with an increasing probability that his opponent records high costs. When θ tends to 0, there is an insignificant gap between the profits recorded by the two firms. If F_1 is neutral it can record a higher profit by 5.7% when F_2 faces high costs or lower by 12.4% when F_2 faces low costs. We can see a significant variation in profit levels only if F_2 has low costs (over 10%).

If the probability to face high cost is close to zero, then we obtain the equilibrium of the Bertrand game in complete information.

Thus, homogeneity of cost assumptions is one of the essential determinants of collusive behaviour. A lower level of cost asymmetry can lead to higher prices in a Bertrand-type competition, which is a valuable antitrust aspect. Also, the size of the cost and demand asymmetry influences the strategies and market share of each firm.

6. Conclusions

The fierce competitive environment has determined the application of game theory models in the process of making optimal price decisions. Seen in this way, the description of economic games through game theory makes it possible to associate mathematical models through which their dynamics are outlined and the modelling of players' behaviour. Manipulation of the information became an important dimension of strategy in price competition, and an important determinant of the market structure, respectively. Being viewed only as a theoretical foundation of analysis of the competition, Bayesian games have become one of the most valuable tools that can be used to analyse present-day problems of Bertrand-type competitive situations.

Unlike the classic Bertrand model in which firms decide on price while seeking to maximize profit, the Bertrand model under incomplete information involves making assumptions about the opponent's costs. The equilibrium strategies (prices) for involved players depend in the Bayesian game on common knowledge cost (for F_1), on private F_2's cost (low cost c_2^m and high cost c_2^M), but also on F_1 beliefs (probabilities) on F_2 costs (θ probability to face c_2^M type and 1- θ probability to face c_2^m type). So, each company's behaviour not only depends on its own type, but is also influenced by possible unknown types and subjective beliefs on competitor's types. If the incomplete information disappears (θ tends to zero or to one) then we found the classical equilibrium strategies in complete information.

A firm can gain a competitive advantage only if its opponent faces high costs. Even if F_1 assumes with a low probability that its opponent faces low costs, the fixed prices are closer to the prices set by the opponent in case it actually records high costs. If F_2 faces low prices, the gap between the prices and profits of the two firms is significant.

In order to understand the complexity of price competition, to outline its dynamics and to model the behaviour of firms in such a competition, further research is needed. In the future studies we will consider the dynamic form of the Bertrand model in incomplete information – like in signalling games, sequential games and last but not least repeated games. In the case of dynamic Bertrand game we will analyse the cooperation strategies versus static Bertrand competition and the deviation (trigger) strategies in finite and infinite repeated games.

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Appendix

Appendix 1. Determine the reaction function for F_2:

i. if F_2 faces high costs

$$\max_{p_2^M} \pi_2^M = (p_2^M - c_2^M)(x_2 - y_2 p_2^M + z_2 p_1)$$

The reaction function of F_2 is determined by the first order condition:

$$\frac{\partial \pi_2^M}{\partial p_2^M} = 0 => x_2 - y_2 p_2^M + z_2 p_1 + (p_2^M - c_2^M) * (-y_2) = 0$$

$$=> x_2 - y_2 p_2^M + z_2 p_1 - y_2 p_2^M + c_2^M y_2 = 0 => p_2^M = \frac{x_2 + z_2 p_1 + c_2^M y_2}{2y_2}$$

$=> p_2^M = \frac{x_2 + c_2^M y_2}{2y_2} + \frac{z_2 p_1}{2y_2}$ reaction function of firm F_2

ii. if F_2 faces low costs

$$\max_{p_2^m} \pi_2^m = (p_2^m - c_2^m)(x_2 - y_2 p_2^m + z_2 p_1)$$

The reaction function of F_2 is determined by the first order condition:

$$\frac{\partial \pi_2^m}{p_2^m} = 0 => x_2 - y_2 p_2^m + z_2 p_1 - y_2 p_2^m + c_2^m y_2 = 0 => p_2^m = \frac{x_2 + z_2 p_1 + c_2^m y_2}{2y_2}$$

$=> p_2^m = \frac{x_2 + c_2^m y_2}{2y_2} + \frac{z_2 p_1}{2y_2}$ reaction function of firm F_2

Appendix 2. Determine the equilibrium price established by firm F_1:

The system formed by the reaction functions of the two firms consists of three equations and three unknowns:

$$\begin{cases} p_2^M = \frac{x_2 + c_2^M y_2}{2y_2} + \frac{z_2 p_1}{2y_2} \\ p_2^m = \frac{x_2 + c_2^m y_2}{2y_2} + \frac{z_2 p_1}{2y_2} \\ p_1 = \frac{x_1 + c y_1}{2y_1} + \frac{z_1[\theta p_2^M + (1-\theta) p_2^m]}{2y_1} \end{cases}$$

The equilibrium price established by firm F_1 is determined by replacing the reaction functions of firm F_2 in the reaction function of firm F_1:

$$p_1{}^* = \frac{x_1 + c y_1}{2y_1} + \frac{z_1\{\theta\left[\frac{x_2 + c_2^M y_2}{2y_2} + \frac{z_2 p_1}{2y_2}\right] + (1-\theta) * \left[\frac{x_2 + c_2^m y_2}{2y_2} + \frac{z_2 p_1}{2y_2}\right]\}}{2y_1}$$

$$p_1{}^* = \frac{x_1 + cy_1}{2y_1} + \frac{z_1\theta x_2 + z_1\theta c_2^M y_2 + z_1 z_2 \theta p_1 + z_1(1-\theta)x_2 + z_1(1-\theta)c_2^m y_2 + z_1 z_2(1-\theta)p_1}{4y_1y_2}$$

$$p_1{}^* = \frac{2x_1y_1 + 2cy_1y_2}{4y_1y_2} + \frac{z_1\theta x_2 + z_1\theta c_2^M y_2 + z_1 z_2 \theta p_1 + z_1 x_2 - z_1\theta x_2 + z_1(1-\theta)c_2^m y_2 + z_1 z_2(1-\theta)p_1}{4y_1y_2}$$

$$p_1{}^* = \frac{2x_1y_1 + 2cy_1y_2}{4y_1y_2} + \frac{z_1y_2[\theta c_2^M + (1-\theta)c_2^m] + z_1z_2[\theta p_1 + (1-\theta)p_1] + z_1x_2}{4y_1y_2}$$

$$p_1{}^*(4y_1y_2 - z_1z_2) = 2x_1y_1 + 2cy_1y_2 + z_1y_2[\theta c_2^M + (1-\theta)c_2^m] + z_1x_2$$

$$p_1{}^* = \frac{2x_1y_1 + 2cy_1y_2 + z_1y_2[\theta c_2^M + (1-\theta)c_2^m] + z_1x_2}{4y_1y_2 - z_1z_2}$$

By replacing the equilibrium price established by F_1 in the reaction functions of F_2, we obtain the equilibrium prices in case F_2, faces high or low costs.